

QBF-BASED SYNTHESIS OF OPTIMAL WORD-SPLITTING IN APPROXIMATE MULTI-LEVEL CELLS

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Motivation

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- Multi-level cells (MLCs)
 - Store 4 bits per cell using 16 discrete levels

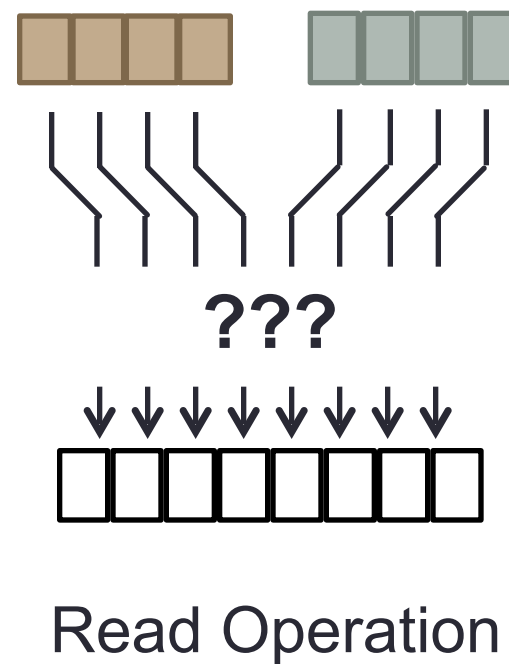
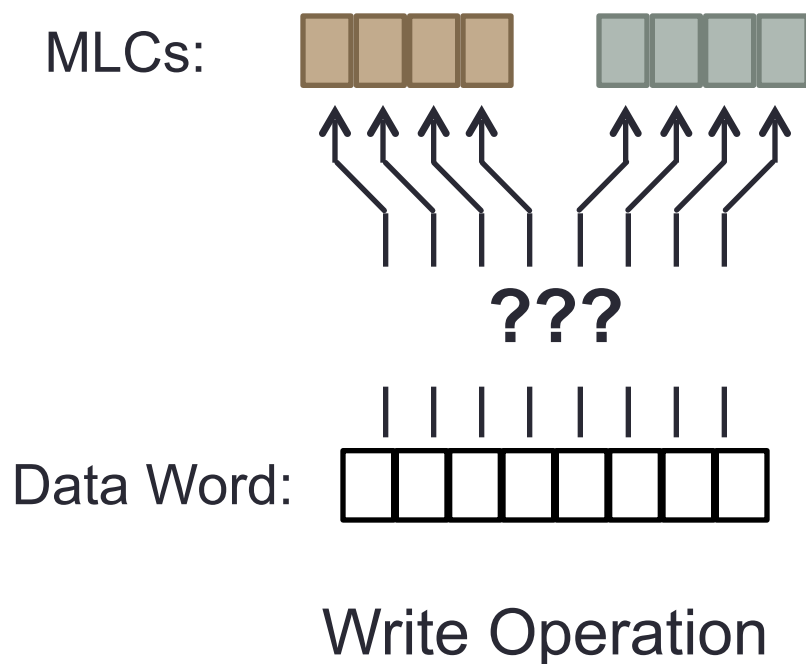
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- Approximate computing
 - Save resources by allowing errors [Sampson et al. '13]
 - Errors between nearby levels
 - But still want to minimize impact of errors

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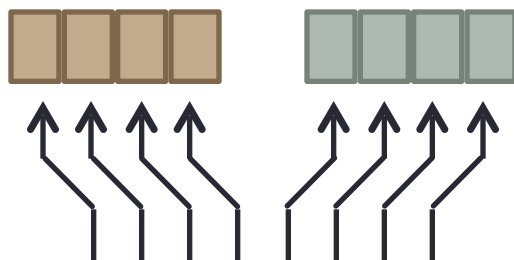
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 - Store 4 bits per cell using 16 discrete levels
- Approximate computing
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 - But still want to minimize impact of errors
- Design decisions
 - Data words don't fit in one MLC
 - How to split word across multiple MLCs?

Example

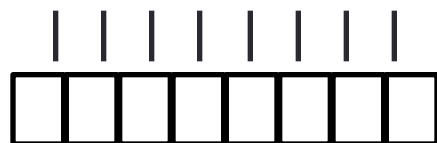


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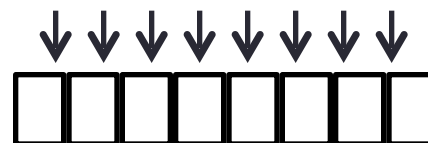
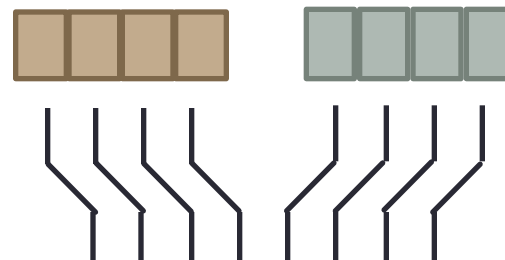
MLCs:



Data Word:

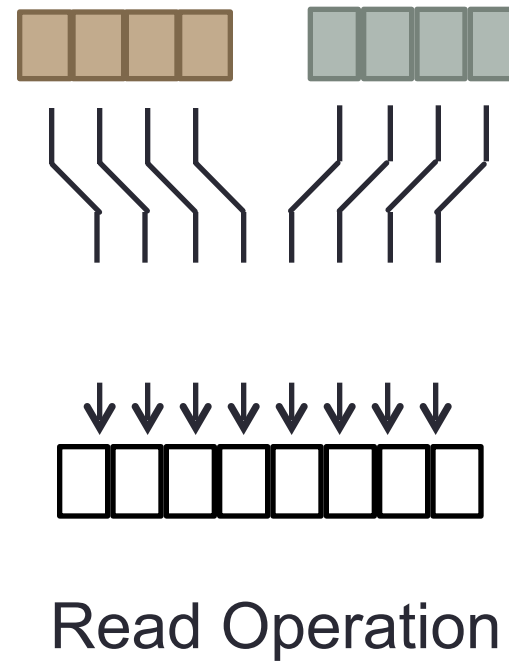
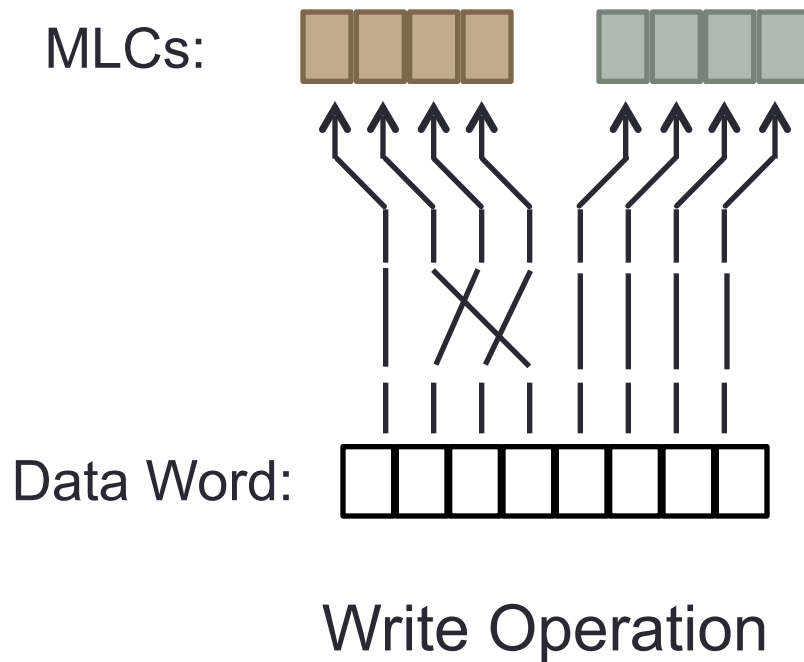


Write Operation

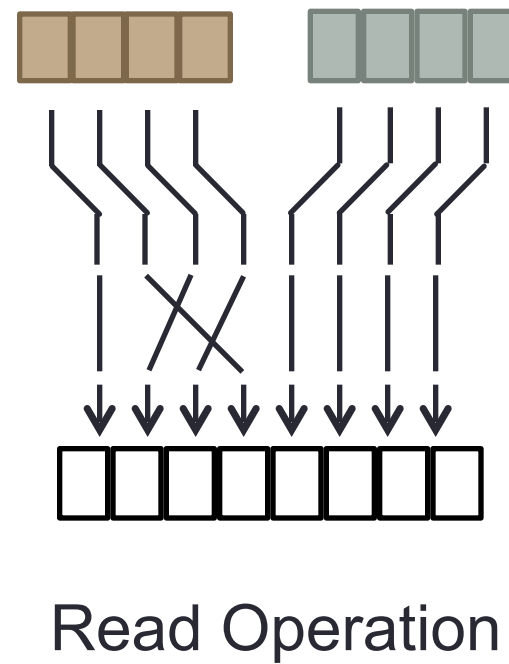
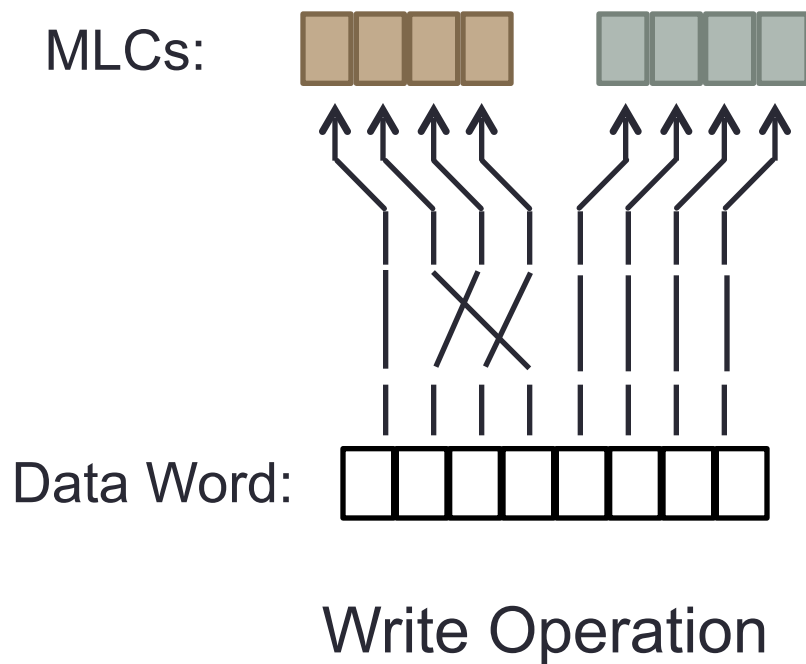


Read Operation

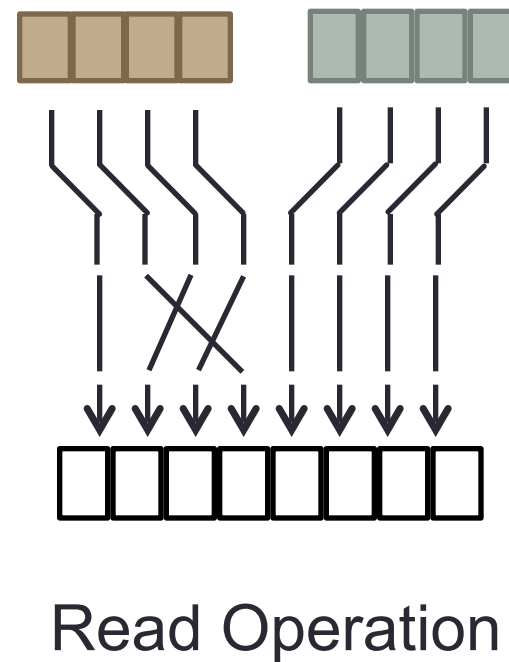
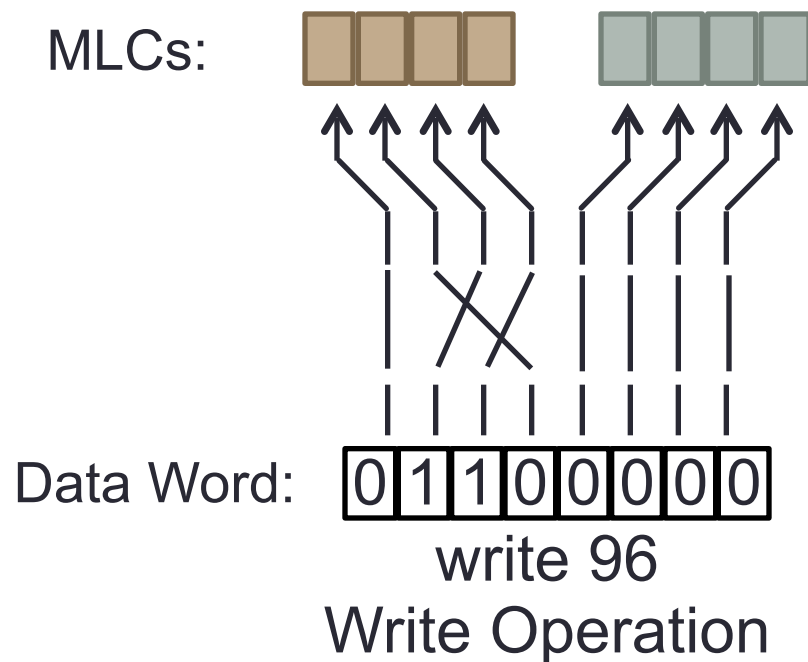
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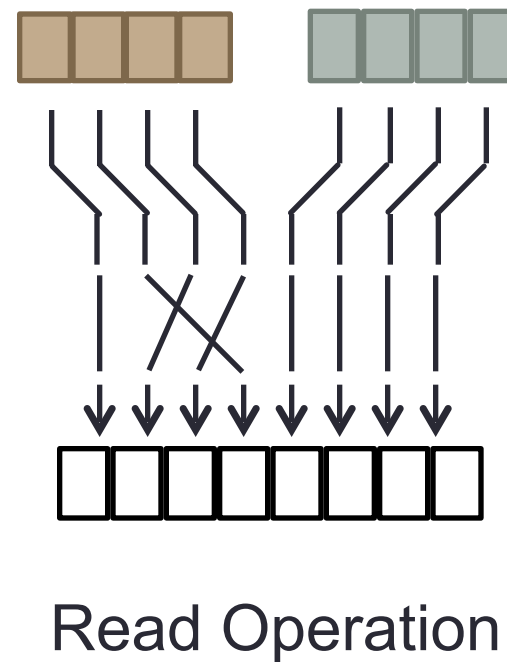
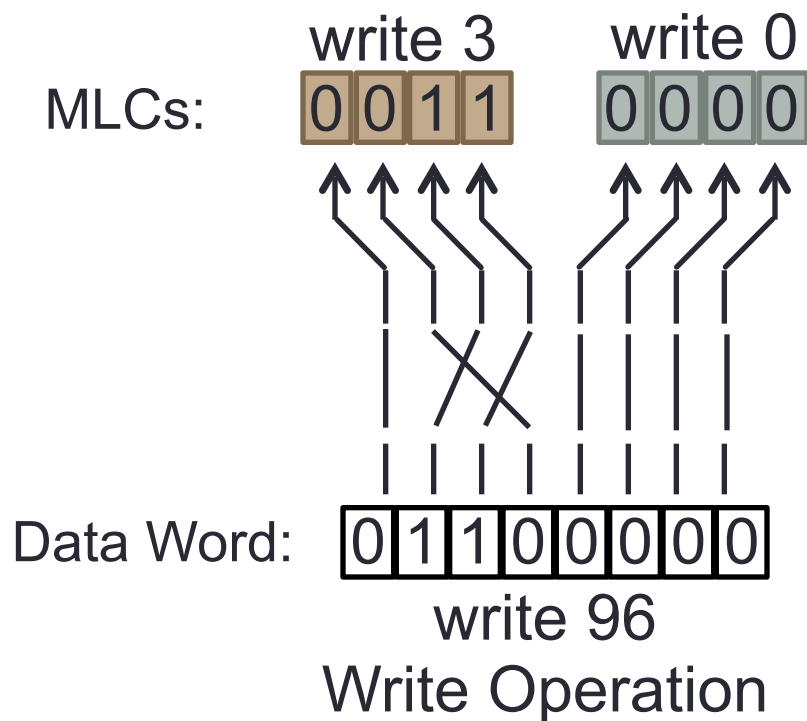
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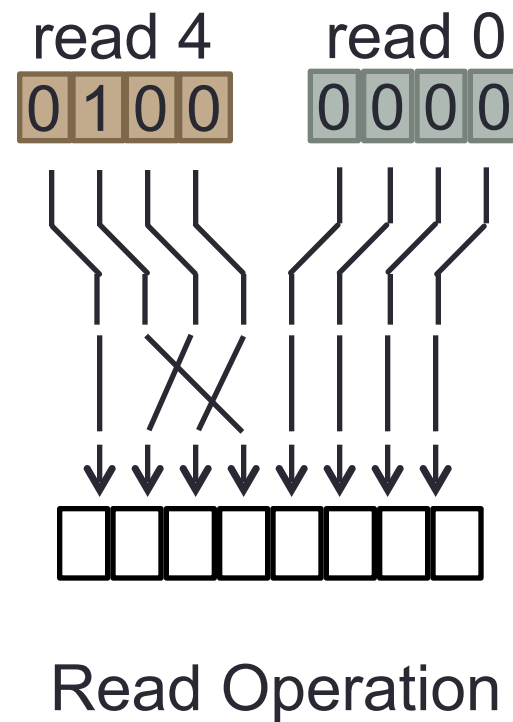
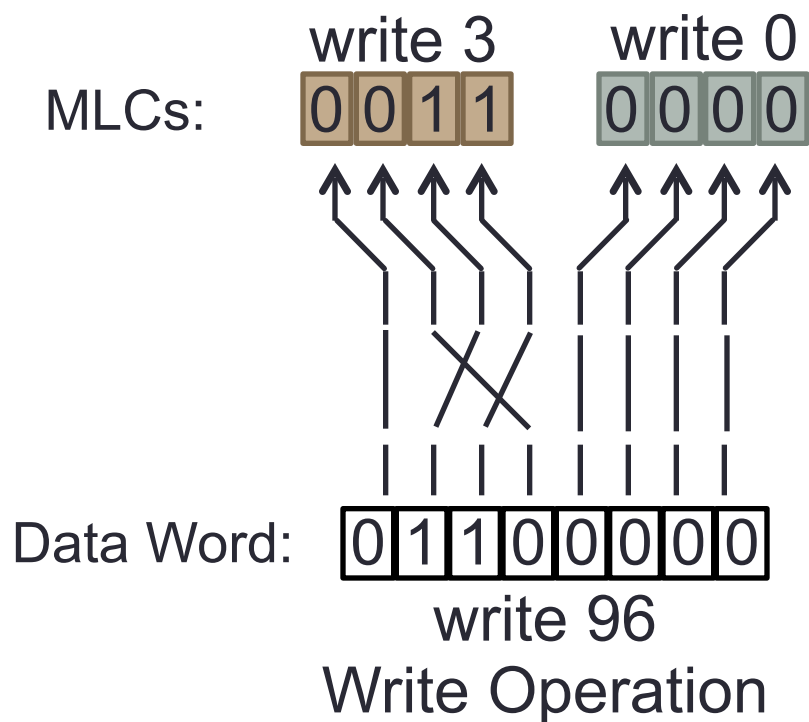
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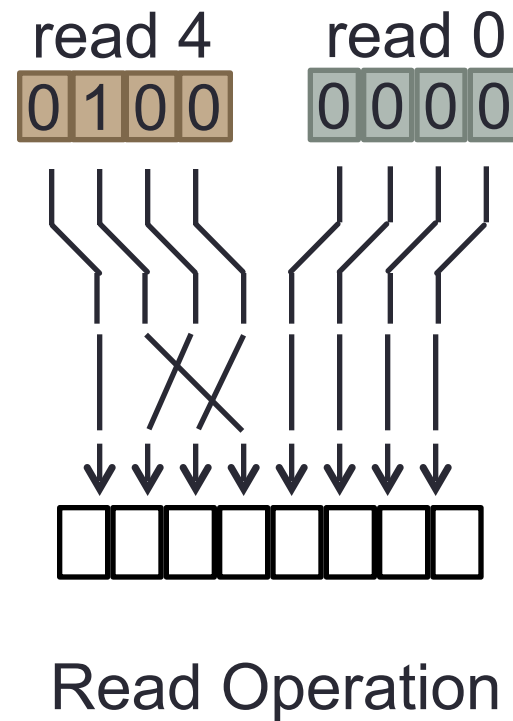
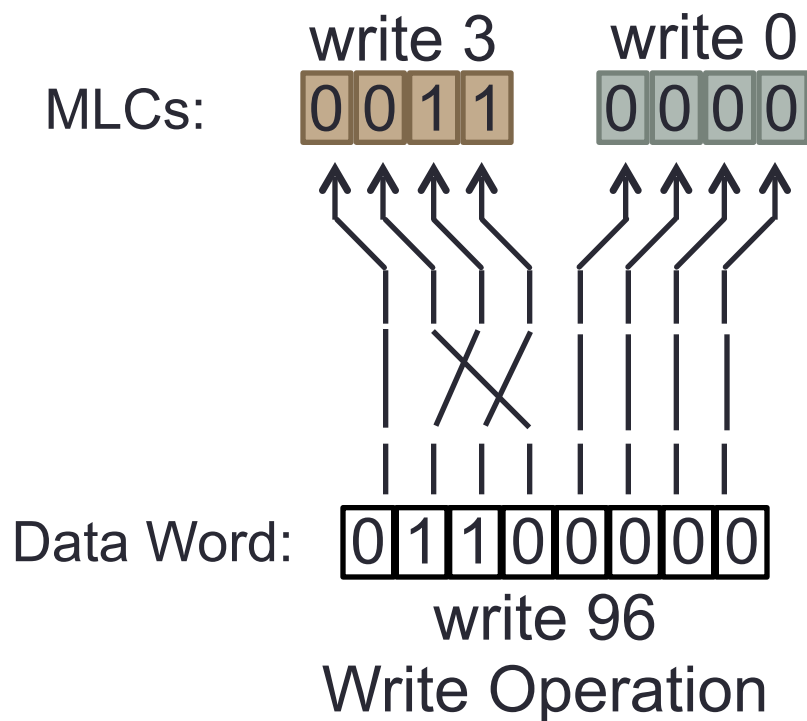


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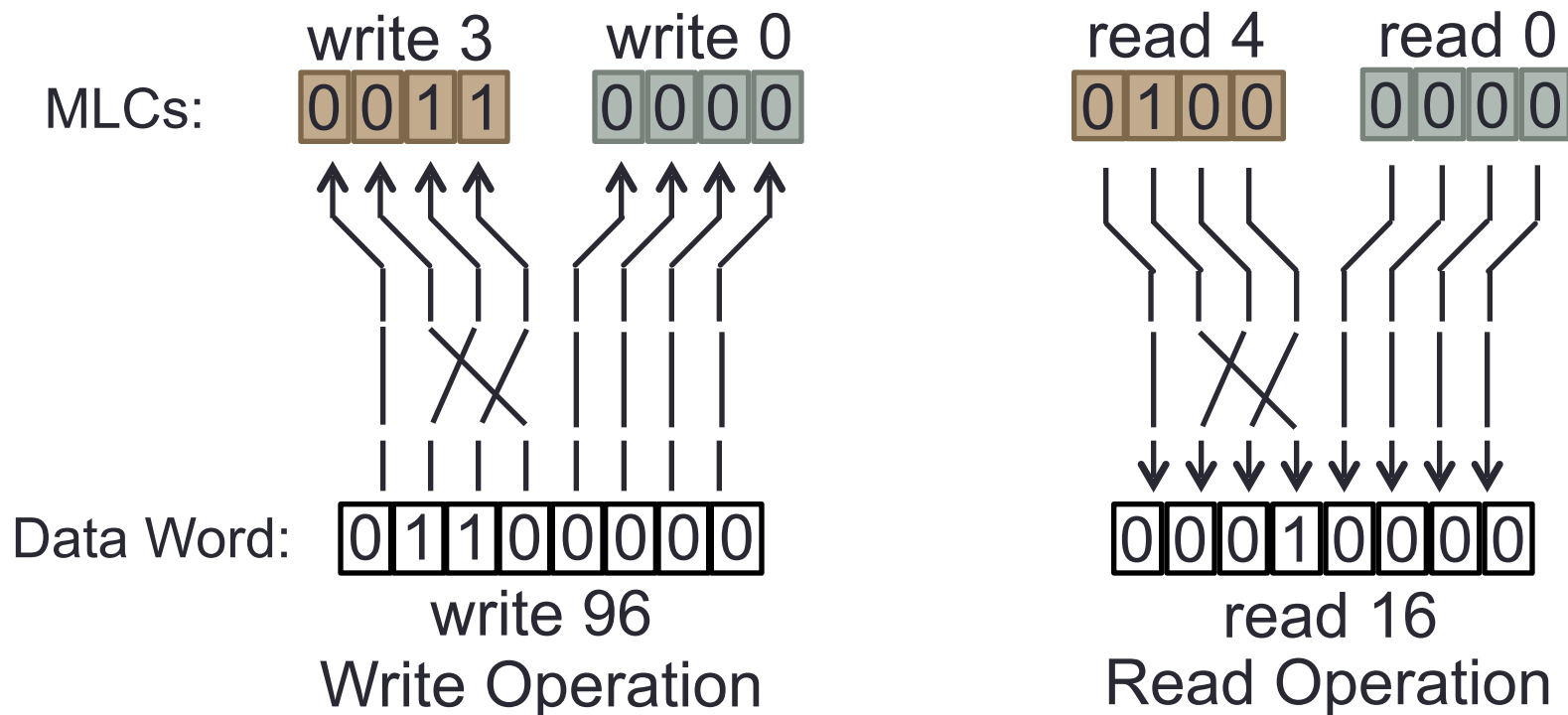
Example

MLC error = 1



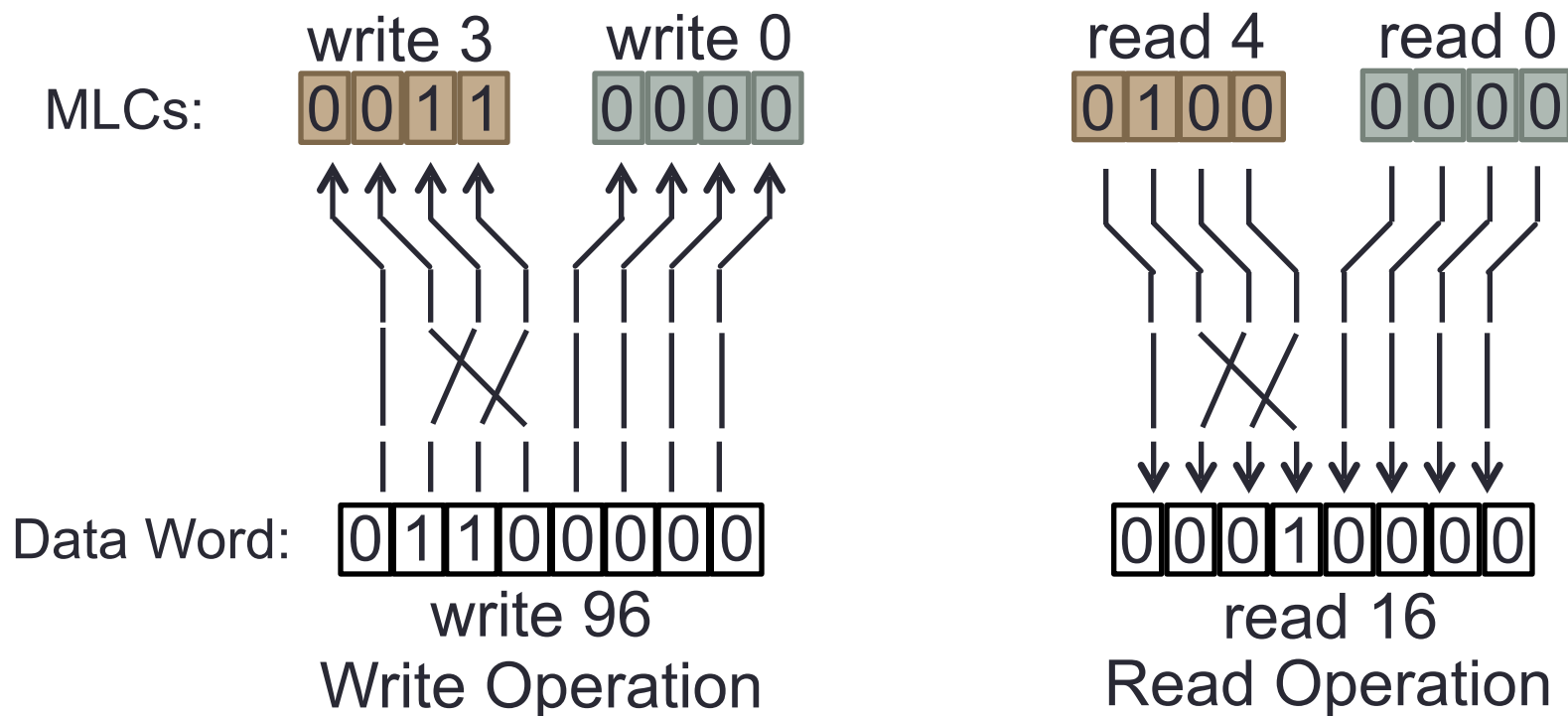
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word error = 80

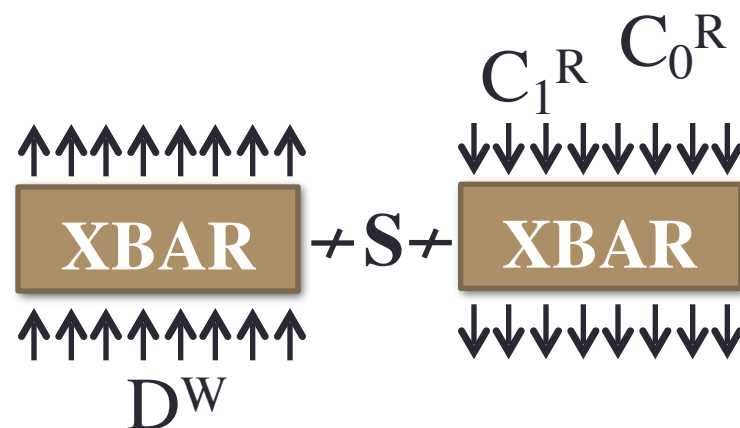
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MLC error = 1

Assuming a fixed upper bound on **MLC error**, can we find a mapping between data word and MLCs that minimizes worst-case **word error**?

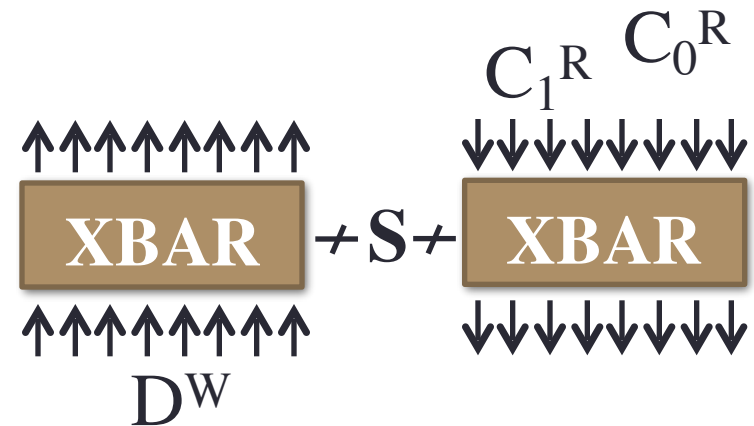
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QBF Formulation of Word-Splitting



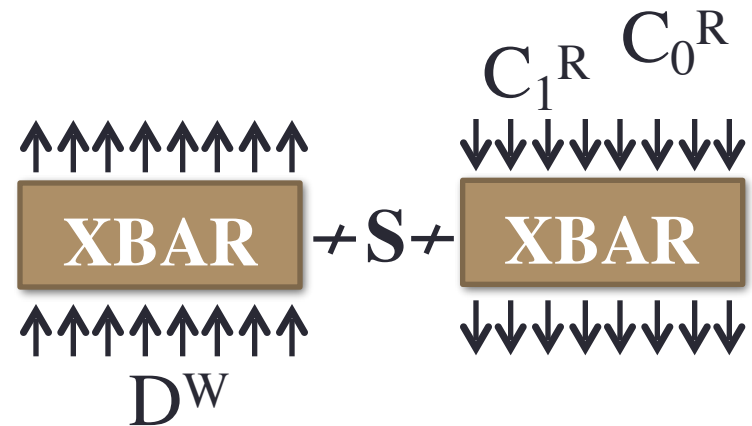
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- QBF: SAT with alternating quantifiers
- Formulate QBF-based synthesis by way of combinational circuit



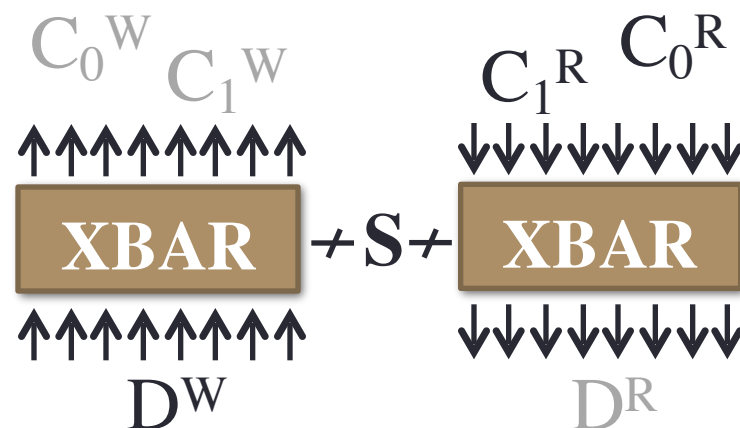
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 - 64 bits specify xbar connections
 - 8-bit written dataword
 - 4-bit value read from each MLC



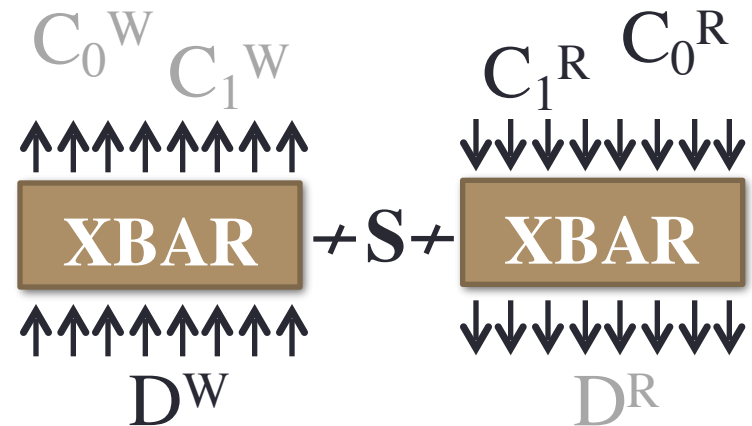
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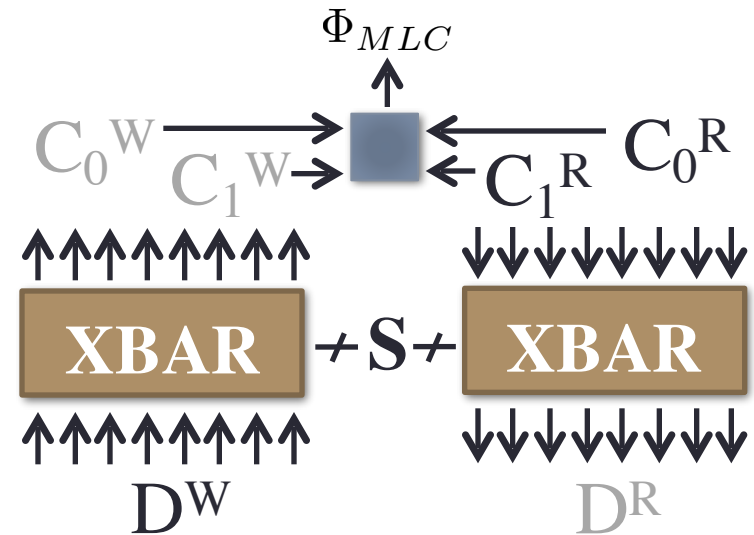
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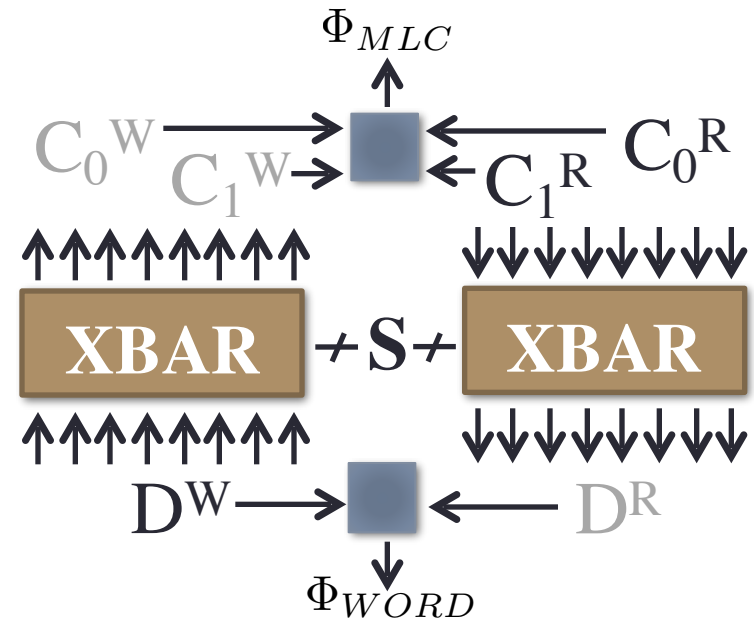
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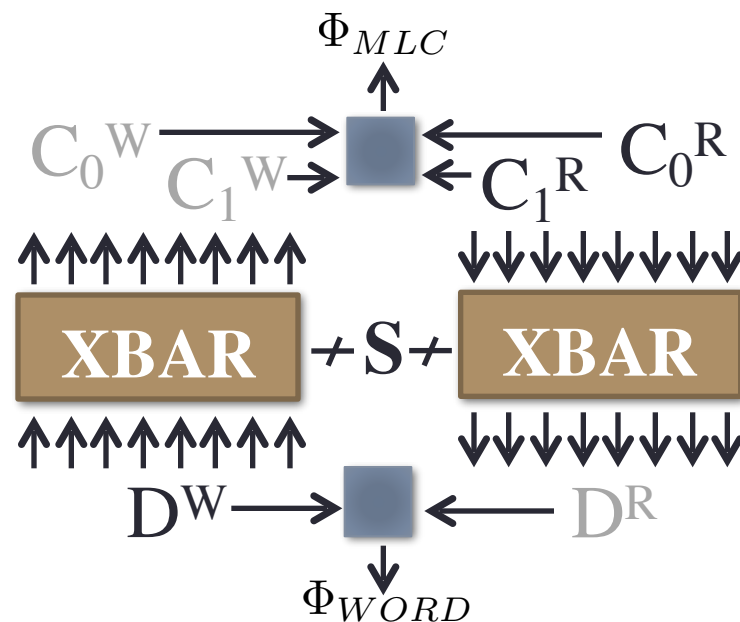


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- Gatewise translate circuit to CNF clauses

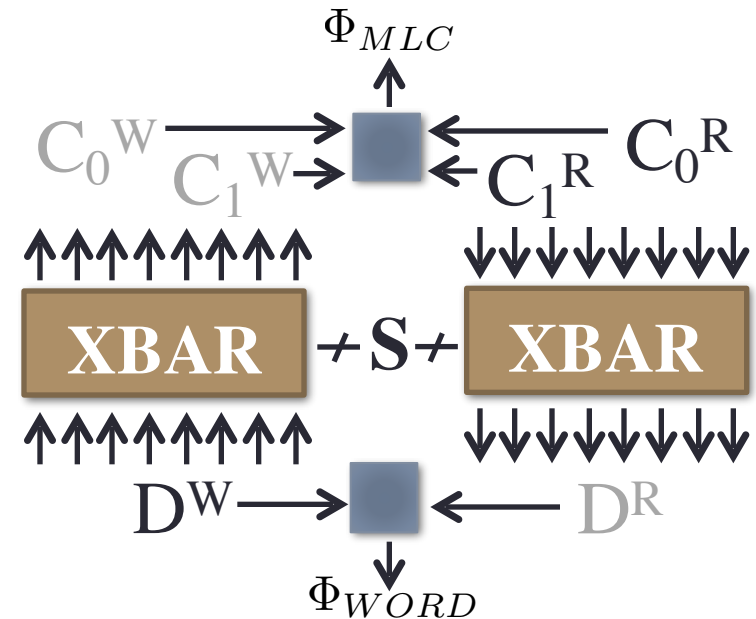


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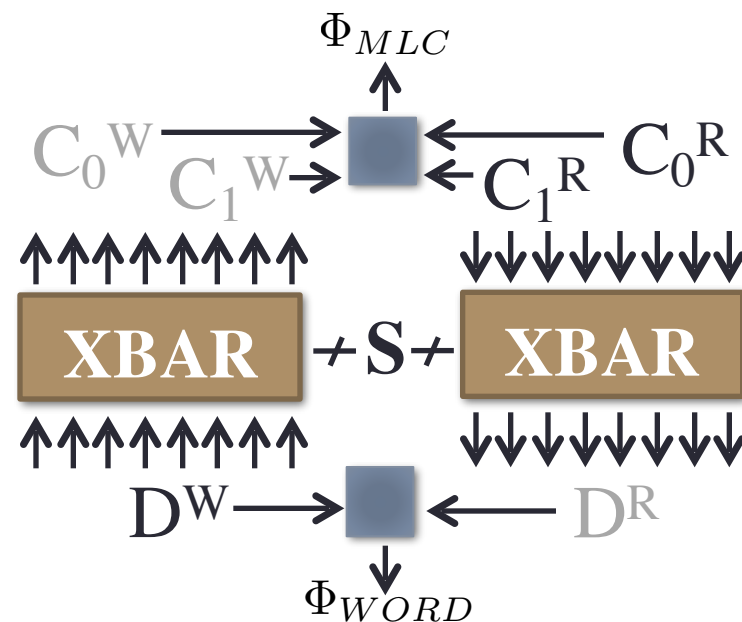


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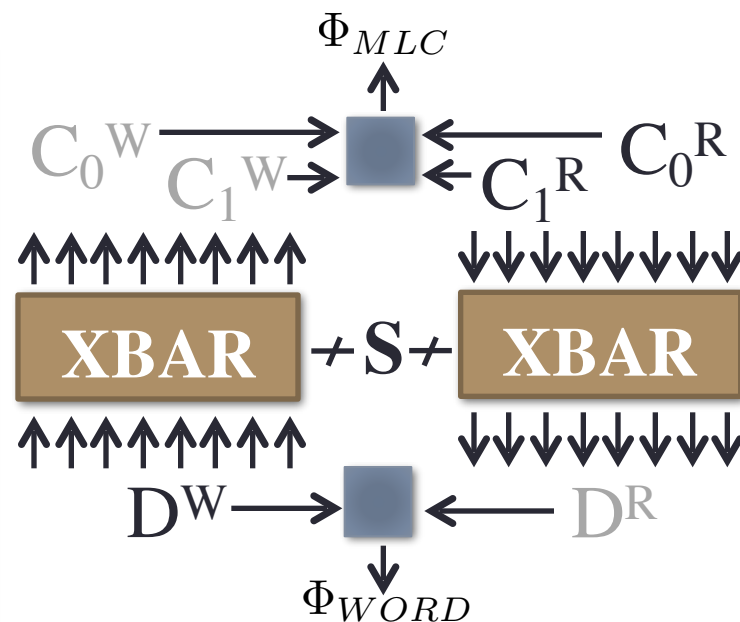
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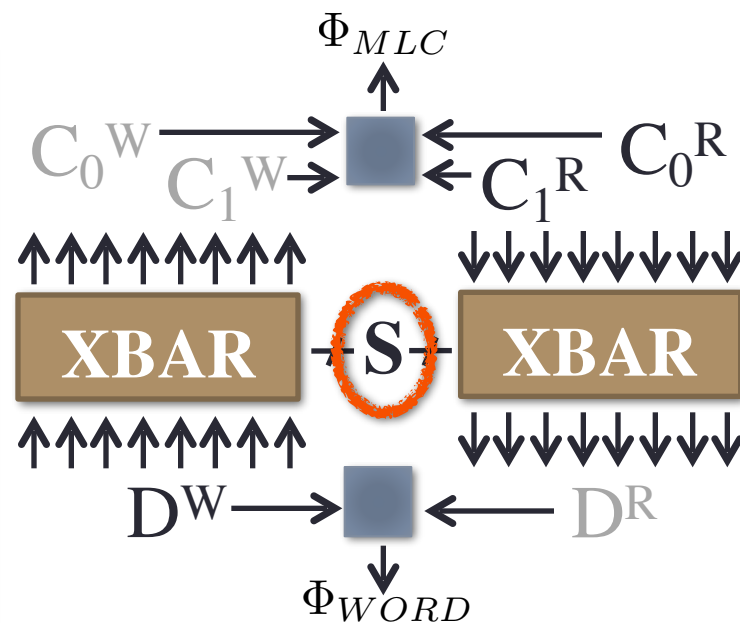
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Results

- QBF-based synthesis to find optimal word-splitting

